Subjective Probability Assessment

Dec 2014

Lesson Objectives

- Terminal Learning Objective: Explain the principles for developing and assessing subjective probabilities
- Enabling Learning Objectives:
  - Demonstrate the imprecision associated with using words to communicate about uncertain quantities
  - Assess subjective probabilities using the fixed probability, fixed value and fractile specification techniques
  - Compare the various heuristics used to assess subjective probabilities and the biases that may result

Words and Expressions

- Exercise

What Is?

The degree of belief or confidence placed in the occurrence of an event by a particular individual based on the evidence available to him or her.

Experts Making Projections

- Long term technological forecasting is difficult
  - Examples
  - "Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $250 and/or six months in prison." (NY State Code of Criminal Procedure)
- Creative ideas or products do not catch on
  - Examples
- So, how does this translate into cost estimating?

Experts

- What do you know (from BCF 106?)
  - Where do you find them?
  - How do you know they are experts?
  - How do you use them?
  - How do you question them?
  - Other……
- How do we use them in CRA's?

Guidelines for Elicitation

- Expert should have a full understanding of the situation before providing estimate (e.g., historical data; complete background)
- You should try to motivate expert and convey feeling of participation and importance in project
- Assist the expert where needed in quantifying their subjective estimates
- Check responses during interview process and ask clarification questions where needed

Guidelines Continued

- Use more than one technique for getting info from the expert if possible to provide a check/balance system
- Try to obtain info from several experts. This will serve as a check
- Consider the background of the expert. You may change your approach depending on this background
- Choose your experts carefully

Some Methods for Obtaining Subjective Probabilities

- Direct Assignment Technique
- The P Technique
- Fixed Interval Technique
- Fractile Specification Technique

Direct Assignment Technique

- Direct assignment of probabilities to discrete events.
- Choices
  - Expert can suggest mutually exclusive events or
  - Analyst can provide expert with events

Once we have the events…

- Have expert assign probabilities to each event such that the probabilities sum to 1
- If the probabilities do not sum to 1 then we have to normalize them as we'll see under the next technique
- Rather than probabilities the expert can provide odds that can be converted to probabilities
- Example

Direct Assignment Example

L.C. Outcomes	Probabilities
95%	.05
95.5%	.10
96%	.15
96.5%	.25
97%	.20
97.5%	.15
98%	.10

Summary: Direct Assignment

- Advantages
  - Simple and easy to apply…remember it is an approach, don't get locked in to rigid step procedure. Adapt.
- Limitations:
  - Works best with discrete events rather than continuous variables. You can use it for continuous variables you just have to treat them as discrete by selecting specific ranges of values for subjective evaluation.

The P Technique

- Similar to direct assignment except expert assigns probabilities in terms of most likely event
- Example: Suppose we have event outcomes x1-x7
  - Step 1: We ask the expert to select the most likely event and assign it a value of P
  - For the purpose of our example the expert told us that x4 was the most likely event

Step 1: P Technique – Select Most Likely Outcome

Outcome	Fraction of "P"
X1	
X2	
X3	
X4	P
X5	
X6	
X7	

Step 2: P Technique – Assign Values

- Next, we ask the expert to assign values to other events in terms of x4 since he said it was the most likely.
- For our example he answers that:
  - x3 and x5 are only half as likely as x4
  - x1 and x7 are only 1/8 as likely as x4
  - x2 and x6 are only ¼ as likely as x4
- Looking at it in terms of a table…….

Step 2: P Technique – Assign values

Outcome	Fraction of "P"
X1	1/8 P
X2	¼ P
X3	½ P
X4	P
X5	½ P
X6	¼ P
x7	1/8 P

Step 3: P Technique – Solve for "P"

- We need to remember that the total probability must add to 1 or said another way,

$$\sum P(X_i)=1$$

- **Solving for P:**

1/8P+1/4P+1/2P+P+1/2P+1/4P+1/8P=1
22/8 P=1 .
P=8/22

Step 4: P Technique – Solve for Probabilities

- Insert P in table - solve for the probabilities.

Event			Prob
X1	1/8(8/22)	=	.05
X2	1/4(8/22)	=	.09
X3	½(8/22)	=	.18
X4	8/22	=	.36
X5	½(8/22)	=	.18
X6	1/4(8/22)	=	.09
x7	1/8(8/22)	=	.05

And if we sum up the probabilities we find they sum to ___?

Summary: P Technique

- Advantage: Simple and easy to administer
- Limitations: This technique, like the direct assignment technique, is designed to estimate discrete parameters. It also requires some knowledge of probabilities on the part of the expert.

Fixed Interval Technique

- Similar to direct assignment technique
- Used with continuous variables
- Step 1: Determine the minimum and maximum values from the expert
- Step 2: Divide this range into intervals, usually of equal size
- Step 3: Expert estimates Prob of variable falling into each interval
- Example

Fixed Interval Example

L.C. Intervals: 95-98%	Probabilities	Cum Prob
95 – 95.49%	.05	.05
95.5 – 95.99%	.10	.15
96 – 96.49%	.20	.35
96.5 – 96.99%	.30	.65
97 – 97.49%	.25	.90
97.5 – 98%	.10	1.00

Summary: Fixed Interval

- Advantages: Simple, easy to employ
- Limitations: Does require some knowledge of probability on the part of the expert

Fractile Specification Technique

- Used with continuous variables
- A technique that poses a question for each of 5 fractiles: the .01, .25, .50, .75, and .99 (think of these as cumulative probabilities)
  - Step 1: Elicit upper and lower bound; the .01 and .99 fractile
  - Step 2. Elicit median; the .5 fractile
  - Step 3. Elicit interquartile values; the .25 and .75 fractiles
- Example

Fractile Specification Example

Order	Fractiles (Cum Prob)	Value
1	.01	
4	.25	
3	.50	
5	.75	
2	.99	

Summary: Fractile Specification Technique

- Advantages: Simple, easy to employ
- Limitations: Does require some knowledge of probability on the part of the expert

Principles for Preparing to Assess Subjective Probabilities

- Choose only uncertain quantities that are important to the analysis
- Clearly define the quantity
- Describe using a scale that is meaningful
- Understand ground rules and assumptions
- Set extreme points first

Heuristic Biases and Errors

- Heuristics (the mental approach used, in this case, to estimate probabilities) that an expert may employ when attempting to provide subjective estimates are a potential source of systematic bias and errors
- We should recognize and understand these

Common Heuristic Biases and Errors

- Availability
  - Expert uses past occurrences of an event to provide the estimate. Tendency to judge more likely if familiar with the event
- Representativeness
  - Belief that likelihood of an event is similar to another outcome based on a superficial similarity (e.g., small sample)
- Anchoring and Adjustment
  - Expert starts with initial value and makes incremental adjustments – inappropriate anchor and insufficient adjustment

Common Heuristic Biases and Errors

- Overconfidence and Overoptimism
  - Tends to assume best possible scenario – exists in estimates and budgets
- Framing Bias
  - People may not make rational choices because of the manner in which information is presented (ask the right question)
- Inexpert expert
  - Rather than referring to right expert, “inexpert” provides opinion to be helpful

Common Heuristic Biases and Errors

- Culture of the Organization
  - Environment in which people work can impact their estimating
- Conflicting Agendas
  - Expert may have vested interest
- Unwilling to Consider Extremes
  - Expert may find it difficult to or unwilling to recognize circumstances that would cause a variable to be extremely low or high.

Common Heuristic Biases and Errors

- Eagerness to Say the Right Thing
  - Occasionally, interviewee will be trying to provide the answer he/she thinks analyst wants to hear
- Units Used in the Estimation
  - Analyst lets expert describe the estimate in units he/she is comfortable with – converts later
- Expert too Busy
  - Can lead to incorrect ranges on variables, etc
- Belief that the expert should be quite certain
  - Large uncertainty not reflective of experts knowledge

Impact of Bound Interpretation

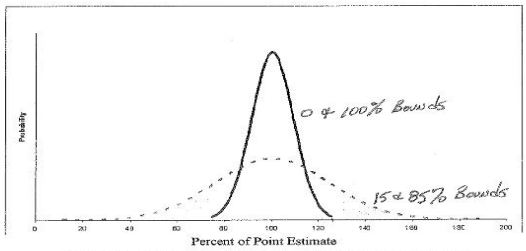

Figure 1 Impact of bound interpretation on a normal distribution

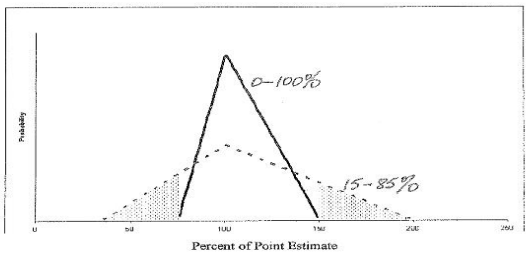

Figure 2 Impact of bound interpretation on a triangular distribution

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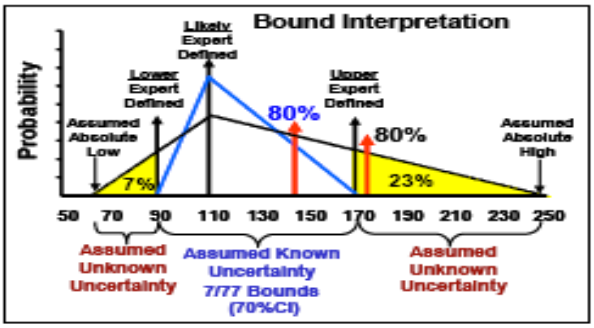
Subjective Uncertainty Bound Interpretation Approach

- Subjective distribution bounds interpreted as capturing 70% of the total uncertainty
  - For symmetrical distributions, lower bound represents 15th percentile and the upper bound represents the 85th percentile
  - For skewed distributions, the 30% uncertainty should be apportioned to the upper and lower bounds according to the ratio of the skewed dist



See Excel file: Specifying Triangular Distribution for solution (NAVAIR)

Subjective Probability Assessment

- Example highlighting elicitation points and techniques

SPA: Final Thoughts

- Questions must be clear
- Perform a dry run
- Have an analyst present during the elicitation
- Prepare brief explanation of the elicitation format and of the model for processing the information
- Avoid coaching
- Session should not be excessively long